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Technical Note

Turbulent forced convection in microtubes

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Abstract

The anomalous and opposing trends in the published data dealing with turbulent flow friction factor and heat transfer coefficient in microchannels, and their apparent disagreement with macroscale correlations, are discussed. It is shown that the modification of turbulent eddy diffusivities, consistent with the way suspended particles may modify turbulence, can explain the observed higher-than-expected heat transfer coefficients in some data. It is thus suggested that suspended microscopic particles may be a major contributor to the aforementioned inconsistencies and disagreements in some of the published data. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Microchannels; Turbulence; Convection; Particles; Heat transfer

1. Introduction

Flow and heat transfer in microchannels with hydraulic diameters of the order of 0.1–1 mm have been studied rather extensively in the recent past [1]. With respect to single-phase flow, different and often opposing trends in experimental data have been reported [2–11]. Acosta [3] measured the momentum, heat and mass transfer for liquid flow in narrow channels with 0.2–0.5 mm-wide gaps, and found good agreement between the measured friction factors and the widely-applied macroscale correlations as long as the channel surfaces were optically smooth. Agreement with macroscale models and laminar flow friction factor data [2,5], and systematic over-prediction of data by macroscale models [8] have also been reported. The published experimental data for turbulent flow, on the other hand, mostly appear to disagree with well-established macroscale models and correlations, for reasons that are not well-understood. The laminar–turbulent flow regime transition appears to be particularly sensitive to the channel size [6]. Measured *Nu* values for turbulent flow of water and methanol in rectangular channels 0.7 mm deep and 0.1–0.8 mm wide, reported in [12–14], were consistently smaller than those predicted by macroscale correlations. An opposite trend, on the other hand, has been reported in [7–10] for turbulent flow in circular microtubes. Adams et al. [10] recently showed that the desorption of dissolved non-condensables may partially be responsible for the observed high heat transfer coefficients in some experiments with water. The qualitative and seemingly baseless argument that the turbulent eddies may be suppressed due to the small channel dimensions when $D_h \lesssim 2$ mm has also been suggested [15] in order to explain lower friction factors and heat transfer coefficients in some data.

The objective of this note is to suggest that suspended microparticles may account for some of the observed disagreements between microchannel data and the well-established macroscale models and correlations.

2. The effect of particles on turbulence

Dispersed particles have been known to influence turbulence in common pipe and channel flows, in view of their apparent drag-reducing effects [16–19]. The main mechanism responsible for the drag-reducing effect of fibrous particles appears to be the suppression of momentum transport within the turbulent core [16–18].

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Notation

B	constant in the logarithmic velocity distribution
D	channel diameter (m)
f	D'Arcy friction factor
h	convection heat transfer coefficient (W/m^2 K)
k	thermal conductivity (W/m K)
Nu	Nusselt number $= hD/k$
Pr	Prandtl number $= \nu/\alpha$
r	radial coordinate (m)
R	channel radius (m)
Re	Reynolds number $= \overline{U}D/\nu$
Re_τ	Reynolds number defined as u^*D/ν
T	temperature (K)
u	velocity (m/s)
$\overline{U}$	channel average velocity (m/s)
u^*	friction velocity $= \sqrt{\tau_w/\rho}$ (m/s)
u^+	dimensionless velocity $= u/u^*$
y	distance for the wall (m)
y_n^+	parameter in Reichardt's model
z	axial coordinate (m)

Greek symbols

α	thermal diffusivity (m^2/s)
ϵ_H	heat transfer eddy diffusivity (m^2/s)
ϵ_M	momentum eddy diffusivity (m^2/s)
η	Kolmogorov's microscale (m)
κ	Karman's constant
μ	dynamic viscosity (kg/ms)
ν	kinematic viscosity (m^2/s)
τ_w	wall frictional stress (Pa)

Subscripts

b	bulk
w	wall
0	constant property

Superscripts

−	average
+	in wall units

Non-spherical particles with relatively small aspect ratios, however, also can modify turbulence, by shifting the occurrence of the laminar-to-turbulent transition to higher Re values. Furthermore, depending on their size, density and concentration, these particles may increase or decrease the friction factor in channels [20,21]. More accurate information about the impact of suspended particles on near-wall turbulence has been provided experimentally [22–25], and by recent direct simulations that account for two-way coupling between particles and the carrier fluid [26,27]. Near-wall turbulent flows are known to be characterized by turbulent coherent structures, which refer to three-dimensional vortices that randomly occur in the turbulent near-wall zone. These structures lead to the approximately cyclic phenomenon of bursting, whereby fluid eddies with high turbulence intensity are released into the main flow, and the sweep-inrush process, whereby blobs of high-turbulence fluid swept from near the wall are replaced with high-momentum fluid entering from upstream. Particles smaller than the Kolmogorov microscale tend to suppress sweeps near walls [27], while large particles tend to enhance turbulence [22,27,28].

For fully-developed, fully-turbulent pipe flow, the Kolmogorov microscale can be estimated from

$$\eta/R \approx Re^{-1/4} Re_\tau^{-1/2}. \quad (1)$$

Particles in the μm size range can thus be comparable to η in microchannels and may influence turbulence. Microscopic particles in the μm and sub-μm size range can often result from imperfect filtration of liquids. Even larger particles can easily find their way into the fluids in experiments, and their complete elimination may not be feasible. Adequate understanding of the effect of microscopic suspended particles on microchannel thermal hydraulics can be very useful and systematic experiments and direct simulation are recommended.

3. Analysis of experimental data

The experimental data of Adams et al. [9,10] are now analyzed in order to demonstrate that a modification of the fully-developed turbulent eddy diffusivities (or, equivalently, the fully-developed turbulent velocity profiles), consistent with the way suspended particles may modify the turbulent velocity profiles, can explain the experimental data trends. These data were all obtained using heated, circular microchannels, with water as the coolant.

For incompressible, fully-developed and constant property flow in a circular tube

$$Re = \frac{\overline{U}D}{\nu} = 4\int_0^{R^+} u^+ \, dy^+ - \frac{4}{R^+}\int_0^{R^+} u^+ y^+ \, dy^+, \quad (2)$$

where $y^+ = yu^*/\nu$; $u^+ = u/u^*$; and

$$u^* = \sqrt{\tau_w/\rho} = \sqrt{f/8}\,\overline{U} = 2\frac{R^+\overline{U}}{Re}. \quad (3)$$

For smooth, circular tubes the fully-developed turbulent velocity profile obeys the well-known law of the wall distribution [29] with $\kappa = 0.4$ and $B = 5.5$ [30], although there exists evidence indicating that κ in pipes may vary with Re [31,32]. Surface roughness and particles, furthermore, modify the velocity profile. The channel friction factor and convection heat transfer coefficient depend on the turbulent velocity profile according to [33]:

$$f = \frac{8R^{+^4}}{Re}\left\{\int_0^{R^+}\left[\int_{r^+}^{R^+}\frac{r^+}{1+(\epsilon_M/\nu)}\,dr^+\right]r^+\,dr^+\right\}^{-1}, \tag{4}$$

$$\frac{k}{hD} = \frac{1}{Nu} = \frac{2}{\left(\overline{U^+}R^{+^2}\right)^2}\int_0^{R^+}\left\{\left(\int_0^{r^+}u^+r^+\,dr^+\right)^2 \times\left[\left(1+\left(\frac{\epsilon_H}{\epsilon_M}\right)Pr\frac{\epsilon_M}{\nu}\right)r^+\right]^{-1}\right\}dr^+, \tag{5}$$

where

$$u^+ = \frac{1}{R^+}\int_{r^+}^{R^+}\frac{r^+\,dr^+}{1+(\epsilon_M/\nu)}, \tag{6}$$

where ϵ_M can be represented using one of several eddy diffusivity models [34–37], e.g., the expression proposed by Reichardt [35]:

$$\epsilon_M/\nu = \kappa\left[y^+ - y_n^+\tanh\left(y^+/y_n^+\right)\right] \tag{7}$$

for $y^+ \leqslant 50$, and

$$\epsilon_M/\nu = (\kappa/3)y^+\left[0.5+(r^+/R^+)^2\right]\left(1+\frac{r^+}{R^+}\right) \tag{8}$$

for $y^+ > 50$, where $\kappa = 0.4$ and $y_n^+ = 11$.

Using the above equations and assuming $\epsilon_H = \epsilon_M$, Petukhov [33] performed extensive computations over the $0.5 \leqslant Pr \leqslant 2000$ and $10^4 \leqslant Re \leqslant 5\times10^6$ ranges, and curve-fitted the results, with an accuracy within only 2%, to an algebraic correlation. To account for the dependence of fluid viscosity on temperature for heating, furthermore, Petukhov derived [33]:

$$(Nu/Nu_0) = (\mu_b/\mu_w)^{0.11}, \tag{9}$$

$$(f/f_0) = (7-\mu_b/\mu_w)/6. \tag{10}$$

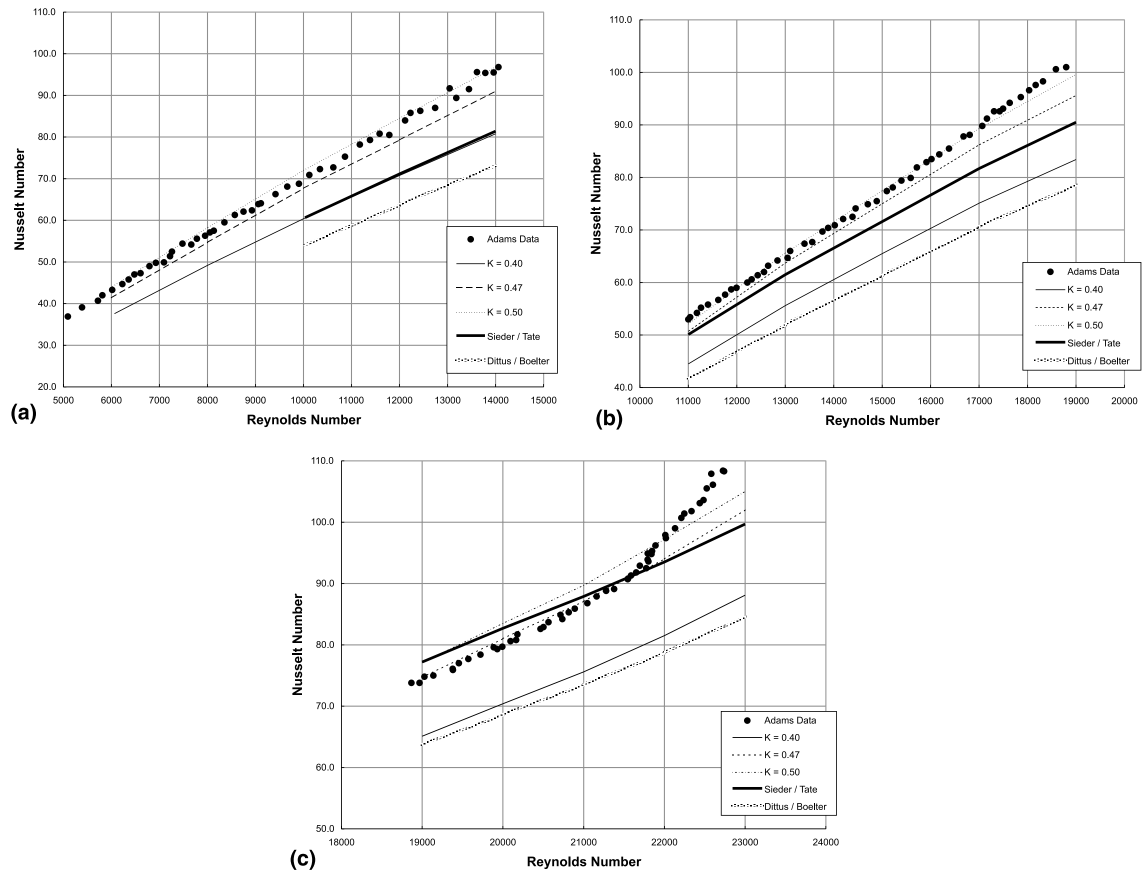

Fig. 1. Comparison between model and data when $y_n^+ = 11$ and κ is adjusted: (a) $q'' = 0.5$ MW/m^2; (b) $q'' = 1.5$ MW/m^2; (c) $q'' = 2.5$ MW/m^2.

Gnielinski [38] has modified Petukhov's correlation in order to extend its applicability to the transition range, $2.3 \times 10^3 < Re < 10^4$, and to short tubes.

In the model calculations presented below, the aforementioned equations were numerically solved according to the following procedure. First an R^+ value is assumed, followed by the definition of a number of radial nodes. Marching from the $r^+ = R^+$ towards the tube center, and using Eqs. (7), (8), and (6) the radial distributions of ϵ_M and u^+ are then found. Using Eq. (2), then Re is obtained, followed by the calculation of $\overline{U^+}$ from Eq. (3). Using Eq. (5), Nu_0 is then calculated, and is used for the estimation of h, and for the calculation of fluid temperature at wall. Eq. (9) is then applied. All numerical integrations are performed using the trapezoidal rule, except for Eq. (6), for which Euler's method is used.

Figs. 1(a)–(c) depict the experimental data of Adams et al. [9,10]. In these carefully-performed experiments, using degassed and deionized water, the heat transfer coefficient near the exit of a uniformly-heated, smooth metallic microtube with $D = 0.76$ mm and 16 cm-long heated segment, were measured. The depicted model calculations were performed by applying the above equations using 1800 radial nodes, with 1000 equal-sized nodes representing the $0.85 \leqslant r/R < 1.0$ zone, and 800 equal-sized nodes representing the remainder of the tube radius. Everywhere $\epsilon_H = \epsilon_M$ was assumed. The model calculations in Figs. 1(a)–(c) were performed by adjusting the value of κ while $y_n^+ = 11$ was maintained. All the data of Adams et al. [9,10], at least for the $Re \lesssim 20\,000$ range, are well-predicted with $\kappa = 0.47$–0.5. Figs. 2(a)–(c) represent model calculations where $\kappa = 0.4$ was maintained, while y_n^+ was adjusted. The bulk of the data points in the $Re \lesssim 20\,000$ can be well-represented with $y_n^+ = 8$–9. The predictions of the correlations of Dittus and Boelter [39] and Sieder and Tate [40] are also shown in the figures. The former correlation under-predicts the data systematically and significantly. The correlation of

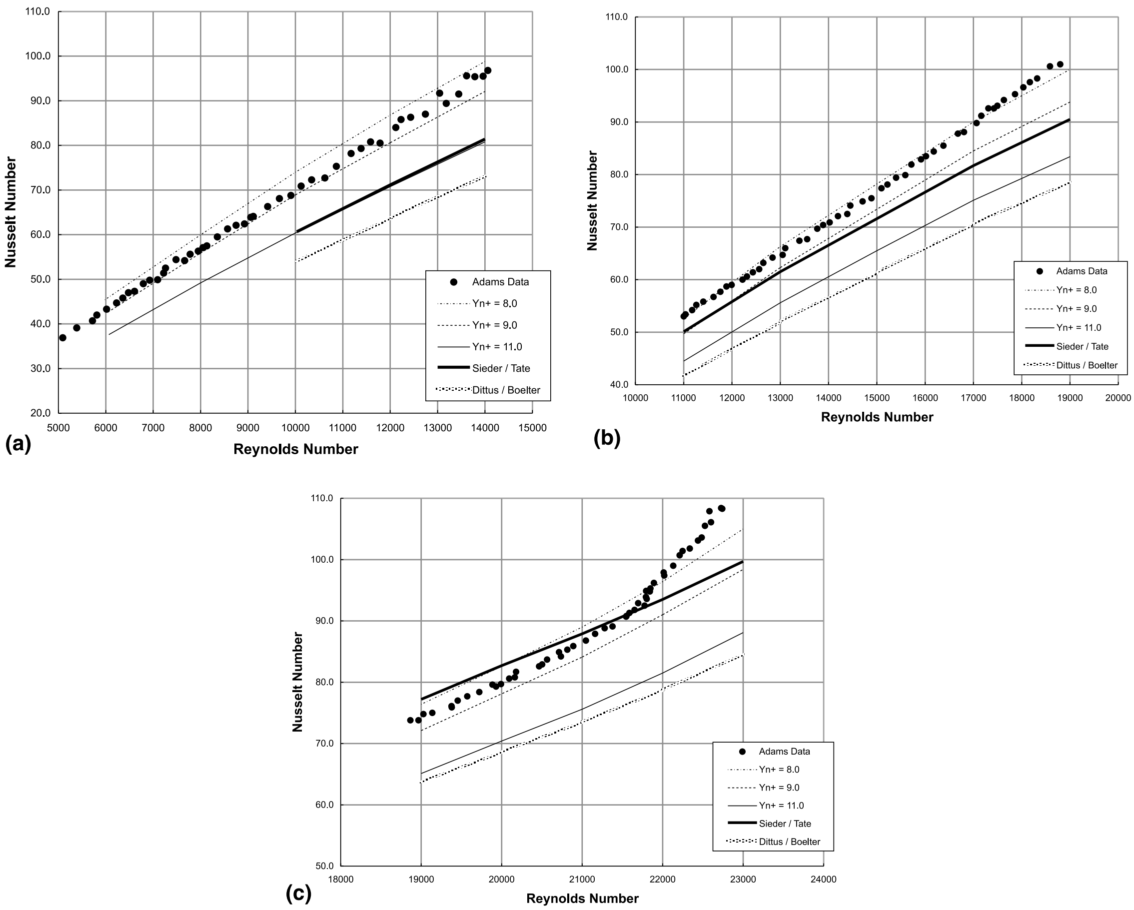

Fig. 2. Comparison between model and data when $\kappa = 0.4$ and y_n^+ is adjusted: (a) $q'' = 0.5$ MW/m^2; (b) $q'' = 1.5$ MW/m^2; (c) $q'' = 2.5$ MW/m^2.

Sieder and Tate under-predicts the data in Figs. 1(a) and (b), but appears to agree with the data in Fig. 1(c) for $Re \lesssim 22\,000$, perhaps by coincidence.

The velocity profile, consistent with the eddy diffusivity expression of Reichardt [35], Eqs. (7) and (8), can be represented as [31]

$$u^+ = \frac{1}{\kappa}\ln(1+\kappa y^+) + 7.4\big[1 - \exp\big(-y^+/y_n^+\big) - \big(y^+/y_n^+\big)\exp(-0.33y^+)\big]. \tag{11}$$

An increase in κ while y_n^+ remains unchanged, as done for the model calculations in Figs. 1(a)–(c), means a flatter core velocity profile, and evidently implies stronger turbulent energy transport in the core, possibly due to turbulence enhancement by large particles. A reduction in y_n^+ while κ remains unchanged, representing Figs. 2(a)–(c), on the other hand, implies stronger damping in the near wall zone, while the core remains relatively unaffected. Either of the two mechanisms can evidently explain the trends in the data of Adams et al. [9,10], and the fact that the data points (with the exception of data representing $Re \gtrsim 22\,000$), which were all obtained in the same test facility, can be represented by essentially the same constants in the eddy diffusivity model lends credibility to the proposed mechanism. A possible explanation for the deviation of model calculations from the data at very high Re values is that with increasing Re, the Kolmogorov microscale is reduced (see Eq. (1)), implying that smaller and potentially more abundant particles become capable of influencing the carrier fluid's turbulence. Values of the Koolmogorov microscale estimated from Eq. (1), are plotted in Fig. 3 for the data of Adams [10], assuming no heating. The aforementioned modifications to the distributions of the eddy diffusivity (and consequently the velocity) would evidently affect the channel friction factor. Friction factors were not measured by Adams et al. [9,10], however.

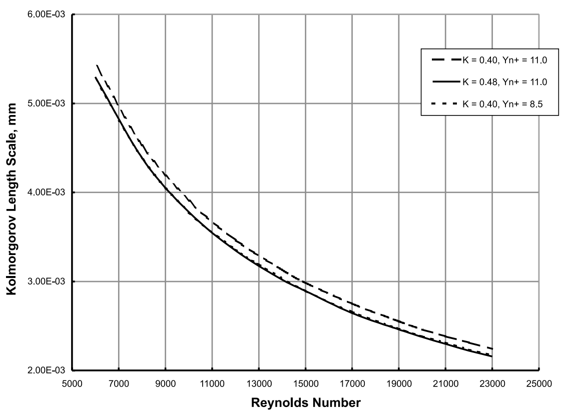

Fig. 3. Estimated Kolmogorov's microscale.

It is worth mentioning that Yu et al. [8] also noted significantly higher heat transfer coefficients than what the common macroscale correlations predicted for their experiments using a 0.102-mm diameter cooled channel. Unfortunately, however, their data appear to represent channel-average measurements, and include entrance effects.

4. Concluding remarks

The experimental data dealing with turbulent flow in microchannels are inconsistent; some indicate higher friction factors and heat transfer coefficients than those predicted by macroscale models and correlations, while others indicate an opposite trend. The origin of these disagreements is not known, and they may result from a multitude of mechanisms. It is suggested that suspended microscopic particles may be a major contributor to the aforementioned inconsistencies and disagreements in some of the published data.

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